# Course of Advanced Automation and Control

January 25, 2019

Surname ____________________ Name ____________________

## Part I - Optimization & Graphs (Prof. D.M. Raimondo)

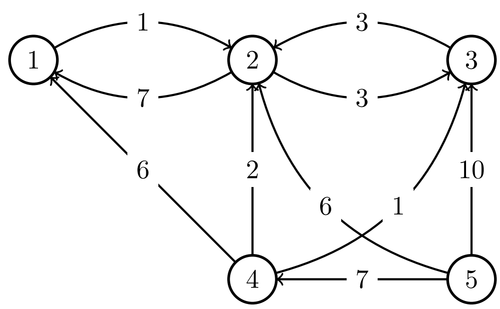
1. Mr. Grecchi has a small shop which is open 5 days a week (Tuesday to Saturday) for 8 hours a day $(9-17)$. He has two employees which can work either with a standard contract or a flexible one. The former provides that the employee will work 8 hours a day per 5 days a week for a cost for the employer of 16€/hour. The latter provides more flexibility: i) the employee does not have to work every day of the week, ii) if its presence is required for a given day of the week, then he has to work at least 4 hours and, in any case, no more than 10, iii) the contract requires at least 30 hours a week and at most 50. The cost of the flexible contract for the employer is of 18€/hour plus a token of 5€ for each of the day the employee is required. A rough estimate of the amount of work required over the week is the following: Tuesday 7 person hours, Wednesday 10 person hours, Thursday 12 person hours, Friday 13 person hours, Saturday 16 person hours. The objective of Mr. Grecchi is to minimize his expenses by choosing the kind of contract and the schedule of the two employees. Please translate the problem into a mixed integer linear program (MILP).

2. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{aligned} \min_{x_1,\delta_1,\delta_2} \quad & -3\delta_1 - \delta_2 + x_1 \\ & 0.5\delta_1 \leq x_1 - 0.25\delta_2 \\ & \delta_1, \delta_2 \in \{0, 1\} \end{aligned}$$

   What is the value of the optimal cost? What is the optimal value of the optimization variables?

3. For the directed network in the figure below, compute all shortest paths from vertex 2 to all other vertices.



4. Consider the mobile robot problem in the figure below. The square in the top right indicates the goal while the black squares the obstacles. Assume the robot can move {up,down,left,right} (no diagonal movements) and that each action has a cost of +1. Assume also that an extra action {stay} is available at the goal only with cost 0.1.

   - What is the minimum horizon required to guarantee the attainment of the goal from any initial condition?
   - For the obtained horizon please formulate the dynamic program (write down the stage cost and the terminal cost) and solve the first step ($J_{N-1}$ and $\mu_{N-1}$) only.
   - Please provide the cost and the optimal input sequence needed to attain the goal starting from the position of the robot given in the figure.